

Viscous Dark Energy and Phantom Field in An Anisotropic Universe

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Abstract

In this paper we have investigated the general form of viscous and non-viscous dark energy equation of state (EoS) parameter in the scope of anisotropic Bianchi type I space-time. We show that the presence of bulk viscosity causes transition of ω^{de} from quintessence to phantom but the phantom state is an unstable state (as expected) and EoS of DE tends to -1 at late time. Then we show this phantom description of the viscous dark energy and reconstruct the potential of the phantom scalar field. It is found that bulk viscosity pushes the universe to a darker region. We have also shown that at late time $q \sim -\Omega^{de}$.

Keywords : Bianchi Type I Model, Dark Energy, Phantom Field

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1 Introduction

There are observational evidences to show that our Universe is undergoing a late-time accelerating expansion and we live in a privileged spatially flat Universe (Perlmutter et al. 1998; Riess et al. 1998; Garnavich et al. 1998; Schmidt et al. 1998; Tonry et al. 2003; Clocchiatti et al. 2006; de Bernardis et al. 2000; Hanany et al. 2000; Spergel et al. 2003; Tegmark et al. 2004; Seljak et al. 2005; Adelman-MacCarthy et al. 2006; Bennett et al. 2003; Allen et al. 2004). These observations indicate that a mysterious type of energy called “dark energy” which is contributing 73% of the total energy of the universe, and approximately 4% baryonic matter and 23% dark matter. However, the observational data are far from being complete (for a recent review, see Perivolaropoulos 2006; Jassal et al. 2005). In fact when the dark energy equation of state (EoS) parameter $\omega^{(de)} = p^{(de)}/\rho^{(de)}$ is less than $-\frac{1}{3}$, the universe exhibits accelerating expansion. The equation of state of dark energy $\omega^{(de)}$ could be equal to -1 (standard Λ CDM cosmology), a little bit upper than -1 (the quintessence dark energy) or less than -1 (phantom dark energy) while the possibility $\omega \ll -1$ is ruled out by current cosmological data (Riess et al. 2004; Astier et al. 2006; Eisentein et al. 2005; MacTavish et al. 2006; Komatsu et al. 2009). There are two main candidates for dark energy (1) cosmological constant (or vacuum energy) and (2) scalar fields. Although, a cosmological constant can explain the current acceleration in a natural way, but would suffer from some theoretical problems such as fine-tuning problem and coincidence problem. Another possible form of dark energy is provided by the dynamically changing DE (Scalar-field dark energy models) including quintessence, K-essence, tachyon, phantom, ghost condensate and quintom, etc. Among these scalar fields, quintessence and phantom are of more scientific interest. Models of dark energy with evolving $\omega^{(de)}$ between $-\frac{1}{3}$ and -1 are refereed to quintessence. But as a candidate for dark energy, quintessence field with $\omega^{(de)} > -1$ is not consistent with the recent observations which indicate that $\omega^{(de)} < -1$ (at $z \sim 0.2$) is allowed at 68% confidence level. Models with $\omega^{de} < -1$ introduce a scalar field ϕ that is minimally coupled to gravity with a negative kinetic energy and are known as “Phantom fields” (Caldwell 2002). Unfortunately, phantom fields are generally plagued by ultraviolet quantum instabilities (Carroll et al. 2003).

The negative pressure of the dark energy may be the cause of the acceleration of the present Universe. However, the nature of the dark energy still remains a mystery. No more than eight years ago, some physicists (McInnes 2002; Barrow 2004) found that, if we assumed the cosmic fluid to be ideal only, i.e. non-viscous, it must bring out the occurrence of a singularity of the universe in the far future. There are two methods to modify or soften the singularity. The first is the effect of quantum corrections due to the conformal anomaly

(Brevik and Odintsov 1999; Nojiri and Odintsov 2003, 2004). The other is to consider the bulk viscosity of the cosmic fluid (Brevik and Hallanger 2004). The viscosity theory of relativistic fluids was first suggested by Eckart (1940), Landau and Lifshitz (1987). The introduction of viscosity into cosmology has been investigated from different view points (Grøn 1990; Padmanabhan and Chitre 1987; Barrow 1986; Zimdahl 1996; Maartens 1996). The astrophysical observations also indicate some evidences that cosmic media is not a perfect fluid (Jaff et al. 2005), and the viscosity effect could be concerned in the evolution of the universe (Brevik and Gorbunova 2005; Brevik et al. 2005; Cataldo et al. 2005). It was also argued in (Zimdahl et al. 2001; Balakin et al. 2003), that a viscous pressure can play the role of an agent that drives the present acceleration of the Universe. The possibility of a viscosity dominated late epoch of the Universe with accelerated expansion has already been mentioned by Padmanabhan and Chitre (1987).

Brevik and Gorbunova (2005), Oliver et al (2011), Chen et al (2011), Jamil and Farooq (2010), Sheykhi and Setare (2010) and Amirhashchi (2013 a,b) have studied viscous dark energy models in different contexts. Recently, viscous dark energy and generalized second law of thermodynamics has been studied by Setare and Sheykhi (2010). Nojiri and Odintsov (2005) studied the effect of modification of general equation of state (EoS) of dark energy ideal fluid by the insertion of inhomogeneous, Hubble parameter dependent term in the late-time universe. They also described several explicit examples of such term which is motivated by time-dependent bulk viscosity or deviations from general relativity. The inhomogeneous term in EoS helps to realize FRW cosmologies admitting the crossing of phantom barrier in a more natural way. Brevik et al. (2010) have also derived a Cardy-Verlinde (CV) formula in FRW universe with inhomogeneous generalized fluid (including viscous fluid). They have also investigated the universality of the dynamical entropy bound near a future singularity as well as near the Big Bang singularity. In the present paper, first we show that the equation of state of dark energy can cross the phantom divided line, $\omega = -1$, by introducing bulk viscosity into the cosmic fluid but this state ($\omega^{(de)} < -1$) is a temporary phase since the viscosity is a decreasing function of time then we suggest a correspondence between the viscous dark energy scenario and the phantom dark energy models in an anisotropic space-time. We show this phantom description of the viscous in the scope of Bianchi type I universe, and reconstruct the potential of the phantom scalar field.

2 The Metric and Field Equations

Although the FLRW models are very successful in explaining the major features of the observed universe but the real universe is not FLRW because of all the structure it contains, and because of the non-linearity of Einstein's field equations the other exact solutions we attain have higher symmetry than the real universe. Thus, in order to obtain realistic models we can compare detailed observations aiming to obtain 'almost FLRW' models representing a universe that is FLRW-like on large scales but allowing for generic inhomogeneities and anisotropies arising during structure formation on a small scale. Such models are given by the so called "Bianchi Type Space-Times" which are homogeneous but anisotropic. Goliath and Ellis (1999) have shown that some Bianchi models isotropise due to inflation.

For the propose of this paper in this section we consider the Bianchi type I space-time in the orthogonal form as

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \quad (1)$$

where $A(t)$, $B(t)$ and $C(t)$ are functions of cosmic time only.

The Einstein's field equations (in gravitational units $8\pi G = c = 1$) read as

$$R_j^i - \frac{1}{2}Rg_j^i = T_j^{(m)i} + T_j^{(de)i}, \quad (2)$$

where $T_j^{(m)i}$ and $T_j^{(de)i}$ are the energy momentum tensors of barotropic matter and dark energy, respectively. These are given by

$$\begin{aligned} T_j^{(m)i} &= \text{diag}[-\rho^{(m)}, p^{(m)}, p^{(m)}, p^{(m)}], \\ &= \text{diag}[-1, \omega^{(m)}, \omega^{(m)}, \omega^{(m)}]\rho^{(m)}, \end{aligned} \quad (3)$$

and

$$T_j^{(de)i} = \text{diag}[-\rho^{(de)}, p^{(de)}, p^{(de)}, p^{(de)}],$$

$$= \text{diag}[-1, \omega^{(de)}, \omega^{(de)}, \omega^{(de)}] \rho^{(de)}, \quad (4)$$

where $\rho^{(m)}$ and $p^{(m)}$ are, respectively, the energy density and pressure of the perfect fluid component or ordinary baryonic matter while $\omega^{(m)} = p^{(m)}/\rho^{(m)}$ is its EoS parameter. Similarly, $\rho^{(de)}$ and $p^{(de)}$ are, the energy density and pressure of the DE component respectively while $\omega^{(de)} = p^{(de)}/\rho^{(de)}$ is the corresponding EoS parameter. We assume the four velocity vector $u^i = (1, 0, 0, 0)$ satisfying $u^i u_j = -1$.

In a co-moving coordinate system ($u^i = \delta_0^i$), Einstein's field equations (2) with (3) and (4) for B-I metric (1) lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\omega^m \rho^m - \omega^{de} \rho^{de}, \quad (5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -\omega^m \rho^m - \omega^{de} \rho^{de}, \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\omega^m \rho^m - \omega^{de} \rho^{de}, \quad (7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = \rho^m + \rho^{de}. \quad (8)$$

If we consider $a = (ABC)^{\frac{1}{3}}$ as the average scale factor of Bianchi type I model then the generalized mean Hubble's parameter H defines as

$$H = \frac{\dot{a}}{a} = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right). \quad (9)$$

The Bianchi identity $G_{ij}^{;j} = 0$ leads to $T_{ij}^{;j} = 0$. Therefore, the continuity equation for dark energy and baryonic matter can be written as

$$\dot{\rho}^m + 3H(1 + \omega^m)\rho^m + \dot{\rho}^{de} + 3H(1 + \omega^{de})\rho^{de} = 0. \quad (10)$$

3 Dark Energy Equation of State

In this section we obtain the general form of the equation of state for the viscous and non viscous energy density ρ^{de} in Bianchi type I space-time when there is no interaction between dark energy density and a Cold Dark Matter (CDM) with $\omega_m = 0$. But before this, we drive the general solution for the Einstein's field equations (5)-(8).

Using the method introduced by Saha (2005), when Eq. (5) is subtracted from Eq. (6), Eq. (6) from Eq. (7), and Eq. (5) from Eq. (7) we obtain

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = 0, \quad (11)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A} \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = 0, \quad (12)$$

and

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{B}}{B} \left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C} \right) = 0. \quad (13)$$

First integral of Eqs. (11), (12) and (13) leads to

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{ABC}, \quad (14)$$

and

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{ABC}, \quad (15)$$

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{k_3}{ABC}, \quad (16)$$

where k_1 , k_2 and k_3 are constants of integration. By taking integral from Eqs. (14), (15) and (16) we get

$$\frac{A}{B} = d_1 \exp[k_1 \int (ABC)^{-1} dt], \quad (17)$$

$$\frac{\dot{B}}{C} = d_2 \exp[k_2 \int (ABC)^{-1} dt], \quad (18)$$

and

$$\frac{\dot{A}}{C} = d_3 \exp[k_3 \int (ABC)^{-1} dt] \quad (19)$$

where, d_1 , d_2 and d_3 are constants of integration.

Now, we can find all metric potentials from Eqs. (17), (19) as follow

$$A(t) = a_1 a \exp(b_1 \int a^{-3} dt), \quad (20)$$

$$B(t) = a_2 a \exp(b_2 \int a^{-3} dt), \quad (21)$$

and

$$C(t) = a_3 a \exp(b_3 \int a^{-3} dt). \quad (22)$$

Here

$$a_1 = (d_1 d_2)^{\frac{1}{3}}, \quad a_2 = (d_1^{-1} d_3)^{\frac{1}{3}}, \quad a_3 = (d_2 d_3)^{-\frac{1}{3}},$$

$$b_1 = \frac{k_1 + k_2}{3}, \quad b_2 = \frac{k_3 - k_1}{3}, \quad b_3 = -\frac{k_2 + k_3}{3},$$

where

$$a_1 a_2 a_3 = 1, \quad b_1 + b_2 + b_3 = 0.$$

Therefore, one can write the general form of Bianchi type I metric as

$$ds^2 = -dt^2 + a^2 \left[a_1^2 e^{2b_1 \int a^{-3} dt} dx^2 + a_2^2 e^{2b_2 \int a^{-3} dt} dy^2 + a_3^2 e^{2b_3 \int a^{-3} dt} dz^2 \right]. \quad (23)$$

In case of non-interacting two fluid the conservation equation (10) for dark and barotropic fluids can be written separately as

$$\dot{\rho}^{de} + 3H(1 + \omega^{de})\rho^{de} = 0, \quad (24)$$

and

$$\dot{\rho}^m + 3H\rho^m = 0. \quad (25)$$

Using Eqs. (14) and (15) in (8), we can write the analogue of the Friedmann equation as

$$\rho = 3H^2 - \sigma^2, \quad (26)$$

where $\rho = \rho^m + \rho^{de}$ is the total energy density and $\sigma^2 = \frac{b_1 b_2 + b_1 b_3 + b_2 b_3}{3a^6}$.

Differentiating Eq. (26) with respect to the cosmic time t , we get

$$\dot{\rho} = 6H\dot{H} - 2\sigma\dot{\sigma}. \quad (27)$$

Using Eqs. (24) and (25) we get

$$\dot{\rho} = -3H(1 + \omega)\rho, \quad (28)$$

where

$$\omega = \frac{\omega^{de}\rho^{de}}{\rho} = \frac{\omega^{de}\rho^{de}}{3H^2\Omega^{de}}, \quad (29)$$

and $\Omega^{de} = \frac{\rho^{de}}{3H^2}$.

On substituting $\dot{\rho}$ from Eq. (27) into Eq. (29) we obtain

$$\omega = -1 - 2 \left(\frac{\dot{H} - Q}{\rho} \right). \quad (30)$$

Using Eqs. (29) and (30), we can rewrite the dark energy equation of state parameter as

$$\begin{aligned} \omega^{de} &= - \left[1 + r + 2 \left(\frac{\dot{H} - Q}{3H^2\Omega^{de}} \right) \right] \\ &= - \left[1 + r + \frac{2}{3\Omega^{de}} (\sigma^2 - (q + 1)) \right], \end{aligned} \quad (31)$$

Here q is the deceleration parameter (see Eq. (49)), $r = \frac{\rho^m}{\rho^{de}}$ and $Q = \frac{\sigma\dot{\sigma}}{3H}$. We note that since always $\dot{\sigma} < 0$ then $Q < 0$. Also since there is no interaction between Dark energy and CDM, r is a decreasing function of time.

Based on the recent observations the deceleration parameter is restricted as $-1 \leq q < 0$. Therefore, from Eq. (31) we observe that the minimum value of ω^{de} which could be achieved for non-viscous DE is -1 i.e EoS of non-viscous DE cannot cross the phantom divided line (PDL) and always varying in quintessence region. Also from this equation we observe that at present time i.e for $r_0 \simeq 0.43$, $\sigma_0^2 \sim 0$, $q_0 \simeq -0.55$, and $\Omega_0^{de} = 0.7$, $\omega_0^{de} \simeq -0.57$. But as mentioned before, according to the current observational data the possibility of $\omega^{de} < -1$ (crossing PDL) is allowed at 66% confidence level. In what follows we show that by assuming a viscous DE, ω^{de} of Eq. (31) crosses PDL i.e there is transition from quintessence to phantom region if viscosity is considered.

In Eckart's theory (1940) a viscous dark energy EoS parameter is specified by

$$p_{eff}^{de} = p^{de} + \Pi. \quad (32)$$

Here $\Pi = -\xi(\rho^{de})u_{;i}^i$ is the viscous pressure and $H = \frac{u_{;i}^i}{3}$ is the Hubble's parameter. On thermodynamical grounds, in conventional physics ξ has to be positive. This is a consequence of the positive sign of the change in entropy as an irreversible process (Landau and Lifshitz 1987). In general, $\xi(\rho^{de}) = \xi_0(\rho^{de})^\tau$, where $\xi_0 > 0$ and τ are constant parameters. A power-law expansion for the scale factor can be achieved for $\tau = \frac{1}{2}$ [49]. It is worth to mention that the Eckart's theory may suffer from causality problem since it only consider the first-order deviation from equilibrium, however, one can still apply it to phenomena which are quasi-stationary, i.e. slowly varying on space and time characterized by the mean free path and the mean collision time.

From Eq. (32) we obtain

$$\omega_{eff}^{de} = \omega^{de} + \frac{\Pi}{\rho^{de}}. \quad (33)$$

Using Eq. (31), above equation can be written as

$$\begin{aligned} \omega_{eff}^{de} &= - \left[1 + r + 2 \left(\frac{\dot{H} - Q}{3H^2\Omega^{de}} \right) \right] - \xi_0 \sqrt{\frac{3}{\Omega^{de}}} \\ &= - \left[1 + r + \frac{2}{3\Omega^{de}} (\sigma^2 - (q + 1)) \right] - \xi_0 \sqrt{\frac{3}{\Omega^{de}}}, \end{aligned} \quad (34)$$

where we have assumed that $\xi(\rho^{de}) = \xi_0 \sqrt{\rho^{de}}$. This is the general form of the viscous dark energy equation of state in Bianchi type-I space-time. From Eq. (34), we observe that $\omega_{eff}^{de} < -1$ (cross PDL) if viscosity is considered. It is obvious that ω^{de} tends to -1 as $\xi(\rho^{de})$ vanishes at late time.

Eq. (34) implies that one can generate phantom-like equation of state from viscous dark energy model in Bianchi type I universe. Thus, we assume that a phantom scalar field ϕ is the origin of the dark energy. Therefore,

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (35)$$

$$p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (36)$$

Thus, ω^{de} is given by

$$\omega_{eff}^{de} = -\frac{V(\phi) + \frac{1}{2}\dot{\phi}^2}{V(\phi) - \frac{1}{2}\dot{\phi}^2}. \quad (37)$$

We observe that in this case $\omega^{de} < -1$. Therefore, according to Eqs. (37) and (34), in the scope of Bianchi type I universe, both non-viscous and viscous dark energy can always be described by phantom. Eqs. (35) and (36) also can be written as

$$V(\phi) = \frac{1}{2}(1 - \omega_{eff}^{de})\rho^{de}, \quad (38)$$

$$\dot{\phi}^2 = -(1 + \omega_{eff}^{de})\rho^{de}. \quad (39)$$

Using ω_{eff}^{de} from Eq. (34) in to Eqs. (38) and (39) we obtain

$$V(\phi) = \frac{3H^2}{2} \left[(r+2)\Omega^{de} - \frac{1}{3} \left(1 + q + \frac{Q}{H^2} \right) + \xi_0 \sqrt{3\Omega^{de}} \right]. \quad (40)$$

$$\dot{\phi}^2 = 3H^2 \left[r\Omega^{de} - \frac{1}{3} \left(1 + q + \frac{Q}{H^2} \right) + \xi_0 \sqrt{3\Omega^{de}} \right]. \quad (41)$$

Now, according to Ref (Alam et al. 2004), we assume the following scalar field equation

$$-\ddot{\phi} - 3H\dot{\phi}^2 + V'(\phi) = 0. \quad (42)$$

The solution of above equation leads to

$$\phi = t, \quad H = f(t), \quad (43)$$

which implies that $f(\phi)$ must satisfy following condition

$$3f(\phi) = V'(\phi). \quad (44)$$

We can define $\dot{\phi}^2$ and $V(\phi)$ in terms of single function $f(\phi)$ also as (Nojiri and Odintsov 2006)

$$V(\phi) = V(\phi) = \frac{3f(\phi)^2}{2} \left[(r+2)\Omega^{de} + \left(\frac{f'(\phi) - Q}{3f(\phi)^2} \right) + \xi_0 \sqrt{3\Omega^{de}} \right], \quad (45)$$

$$1 = 3f(\phi)^2 \left[r\Omega^{de} + \left(\frac{f'(\phi) - Q}{3f(\phi)^2} \right) + \xi_0 \sqrt{3\Omega^{de}} \right]. \quad (46)$$

From Eq. (46) we can find Ω^{de} as

$$\Omega^{de} = \frac{-3\xi_0^2 + \sqrt{9\xi_0^4 + 4r^2 \left(\frac{1-f'(\phi)+Q}{3f(\phi)} \right)^2}}{2r^2}. \quad (47)$$

Substituting the above Ω^{de} into Eq. (45), we obtain the scalar potential as following

$$V(\phi) = \frac{3f(\phi)^2}{2} \left[\left(\frac{r+2}{2r^2} \right) \left(-3\xi_0^2 + \sqrt{9\xi_0^4 + 4r^2\Gamma^2} \right) + \frac{1}{3f(\phi)^2} - \Gamma + \sqrt{1.5} \frac{\xi_0}{r} \sqrt{-3\xi_0^2 + \sqrt{9\xi_0^4 + 4r^2\Gamma^2}} \right], \quad (48)$$

where $\Gamma = \frac{1-f'(\phi)+Q}{3f(\phi)^2}$.

For completeness, we give the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}, \quad (49)$$

which combined with the Hubble parameter and the dimensionless density parameters form a set of useful parameters for the description of the astrophysical observations. From eqs. (26)-(28) and (30), we obtain

$$\frac{\dot{H}}{H^2} = \frac{1}{3} \frac{\sigma\dot{\sigma}}{H^2} + 2(1+r)(\dot{H} - \sigma\dot{\sigma})\Omega^{de}. \quad (50)$$

Using Eq. (50) in Eq. (49), we get

$$q = - \left[(1+r)\Omega^{de}(1+2\dot{H}-2\sigma\dot{\sigma}) + \frac{\sigma(\sigma+\dot{\sigma})}{3H^2} \right]. \quad (51)$$

Above equation shows that at late time, $q \sim -\Omega^{de}$.

4 Late Time Geometry of The Model

From geometrical point of view, all FLRW based cosmological models are homogeneous and isotropic. It is clear that such models can not describe the evolution of our universe in it's early times where, geometrically, it was inhomogeneous. Also, according to the recent observations, there is tiny variations between the intensities of the microwaves coming from different directions which means that our current universe is anisotropic. Moreover, as far as we use the maximally symmetric FLRW metrics, one can always ask: does the universe necessarily have the same symmetries on very large scales outside the particle horizon or at early times? Hence, to be more general, it is quiet reasonable to use generalized FLRW equations by considering an anisotropic metric (Bianchi Models).

To show that how Bianchi models tend to isotropy, we define the generalized mean Hubbles parameter H as

$$H = \frac{1}{3} (H_1 + H_2 + H_3), \quad (52)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubbles parameters in the directions of x , y , and z respectively. The mean anisotropy parameter A_m is given by

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2, \quad (53)$$

where $\Delta H_i = H - H_i$. Using Eqs. (20)–(22) and (52) in Eq. (53), we obtain

$$A_m = \frac{1}{3} K \frac{a^{-6}}{1 + a^{-6}}, \quad (54)$$

where $K = b_1^2 + b_2^2 + b_3^2$. Since $a = (1+z)^{-1}$, we can re-write the above equation in terms of redshift z as

$$A_m = \frac{1}{3} K \frac{(1+z)^6}{1 + (1+z)^6}. \quad (55)$$

Equation (55) obviously shows that at late time i.e $z \rightarrow -1$, $A_m \rightarrow 0$. Also since for $K = 0$ (i.e $b_1 = b_2 = b_3 = 0$) our model is equivalent to the FLRW model as from Eq. (55), we obtain $A_m = 0$.

5 Conclusion

Models with ω^{de} crossing -1 near the past have been mildly favored by the analysis on the nature of dark energy from recent observations (for example see Astier et al. 2006). SNe Ia alone favors a ω larger than -1 in the recent past and less than -1 today, regardless of whether using the thesis of a flat universe (Astier et al. 2006; Nojiri and Odintsov 2006) or not (Dicus and Repko 2004). In this paper, we have studied the possibility of crossing phantom divided line ($\omega^{de}=-1$) in the scope of anisotropic Bianchi type I space-time. The general form of the EoS parameter of viscous and non-viscous dark energy has been investigated. It is found that the presence of bulk viscosity causes transition of ω^{de} from quintessence to phantom. But since $\xi(\rho^{de}) = \xi_0 \sqrt{\rho^{de}}$ and ρ^{de} is a decreasing function of time in an expanding universe we conclude that the bulk viscosity dies out as time goes on. In another words, the phantom state is an unstable state (as expected) and EoS of DE tends to -1 at late time. It is worth to mention that equations (5)–(8) can be recast in terms of H , Σ and q as

$$\bar{p} = H^2(2q - 1) - \Sigma^2, \quad (56)$$

$$\rho = 3H^2 - \Sigma^2. \quad (57)$$

Here Σ^2 is the shear scalar which is given by

$$\Sigma^2 = \frac{1}{2}\Sigma^{ij}\Sigma_{ij}, \quad (58)$$

where

$$\Sigma_{ij} = u_{i;j} + \frac{1}{2}(u_{i;k}u^k_{j} + u_{j;k}u^k_{i}) + \frac{1}{3}\theta(g_{ij} + u_i u_j).$$

From equations (57)-(58), we obtain

$$\frac{\ddot{a}}{a} = \frac{1}{2}\xi\theta - \frac{1}{6}(\rho^{de} + 3p^{de}) - \frac{1}{6}(\rho^m + 3p^m) - \frac{2}{3}\Sigma^2, \quad (59)$$

which is Raychaudhuri's equation for given distribution. Above equation can be written as

$$\frac{\ddot{a}}{a} = \frac{1}{2}\xi\theta - \frac{1}{6}\rho^{de}(1 + 3\omega^{de}) - \frac{1}{6}\rho^m(1 + 3\omega^m) - \frac{2}{3}\Sigma^2. \quad (60)$$

Equation (59) shows that for $\rho^{de} + 3p^{de} = 0$, acceleration is initiated by bulk viscosity only. In absence of bulk viscosity dark energy contributes the acceleration (since $\omega^{de} < -1$, then $1 + 3\omega^{de} < -1$, i.e the second term in the right hand side of Eq. (60) is always positive). Therefore, the presence of bulk viscosity pushes the universe to a darker region.

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